

HRTFs from Point Source Representations

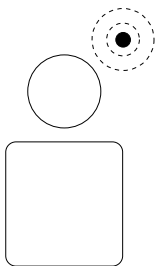
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Ambisonics Symposium '09 Graz

Motivation

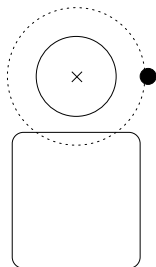
- The aim is to improve quality of near-field HRTF synthesis.
- Existing methods: HRTFs selected by parallax, and modified by the analytic sphere model.
- These ignore response of other features to curvature, pinna etc.



Fourier-Bessel expansion

Use FBE / HOA to represent a region of a point-source soundfield around the head.

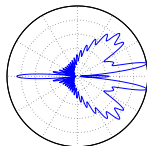
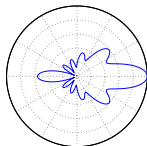
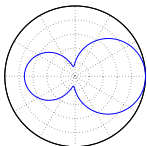
Simplest case:



Region should enclose important scattering surfaces.

Region radius $r \approx M/k$

Fourier-Bessel expansion



$$M = kr_s = 1, 5, 20$$

- Significant reverse energy, greater at low M .
- $F_m(kr_s)$ awkward. Can be limited, preserving phase in pass band.

Fourier-Bessel expansion

$$H(\mathbf{r}, k) = \sum_{i,m,n} D_{imn} F_m(kr) Y_{mn}(\hat{\mathbf{r}}) H_i(k) \quad (1)$$

$$H(\mathbf{r}, k) = \sum_m F_m(kr) H_m(\hat{\mathbf{r}}, k) \quad (2)$$

$$H_m(\hat{\mathbf{r}}, k) = \sum_n Y_{mn}(\hat{\mathbf{r}}) H_{mn}(k) \quad (3)$$

$$H_{mn}(k) = \sum_i D_{imn} H_i(k) \quad (4)$$

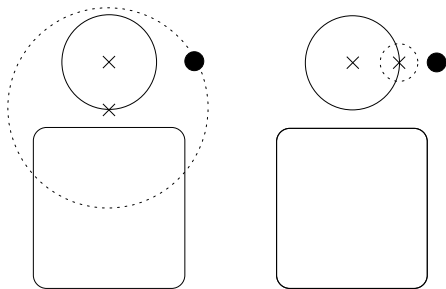
$\{H_i\}$ defines the centre of the reconstruction.

D_{imn} is a pseudo-inverse found using regularization Tychonov, TSVD, etc.

$\{H_i\}$ hole, noise, alignment.

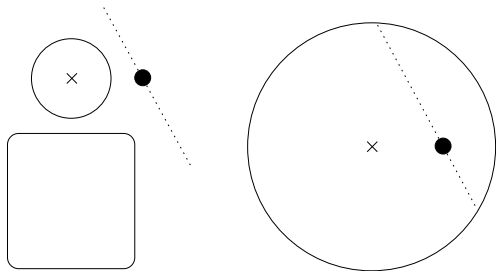
Shifted Point-source FBE

$$H_i \rightarrow H_i e^{i\hat{k}_i \cdot \mathbf{r}_t}$$

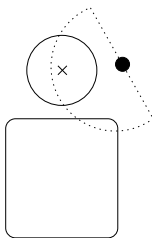


- H_i precision shifted to ear centre not the same as truncation.

Shifted Point-source FBE limit



- Speaker region : create a point-source half-space.



- + H_i hole less of a problem.
- + Wide angle at low order : closer.
- + Independent of kr : no F_m , no colouration at low order.
- + No boundary limit, without extra shifting.
- Limited wave-near-field even at high order : Sphere model.
- No far-field.

Focused source : theory

What is the error of a focused source in 3D, 2D?
Sommerfield-Weyl:

$$\frac{1}{r} e^{ikr} = \frac{i}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{1}{k_z} e^{ik_x x + ik_y y + ik_z |z|} dk_x dk_y \quad (5)$$

Rearrange as focused source: $\frac{1}{r} e^{ikr}, z < 0$ $\frac{1}{r} e^{-ikr}, z > 0$

k_z real : Planewaves, the focused source.

k_z imaginary : The error, exponentially decaying half spaces about $z = 0$, non-smooth.

Analytic error on z-axis

In 3D:

$$\frac{i}{2\pi} \int_k^\infty \frac{1}{k_z} e^{ik_z z} 2\pi k_{xy} dk_{xy} = 1/z \quad (6)$$

Error is the same magnitude as the 3D point source!

In 2D:

$$\frac{1}{\pi} \int_0^\infty \frac{1}{k_x} e^{ik_y y} dk_y \sim 1/z \quad (7)$$

Error decays faster than the 2D point source $\sim z^{-1/2}$

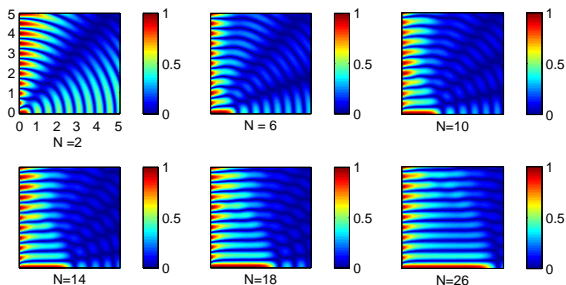
Using HOA conventions:

$$B_{mn} = \frac{-i}{2\pi} \int_{\hat{\mathbf{k}} \cdot \hat{\mathbf{z}} > 0} Y_{mn}(-\hat{\mathbf{k}}) d\Omega \quad (8)$$

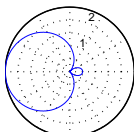
$$= -i(-1)^m \sqrt{2m+1} \int_0^1 P_m(z) dz \quad (9)$$

$$= -i\sqrt{2m+1}(-1)^{m-1/2} \frac{m!!}{m(m+1)(m-1)!!} \quad (10)$$

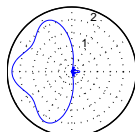
Absolute relative error of FS compared with PS:



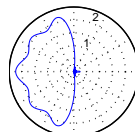
- Error on z-axis clear.
- In reconstruction, zero higher orders.
- Best compromise for $M = 6..10$



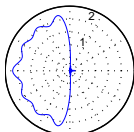
N=2



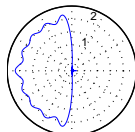
N=6



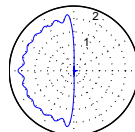
N=10



N=14



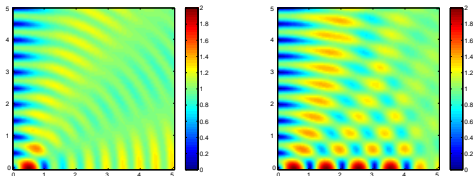
N=18



N=26

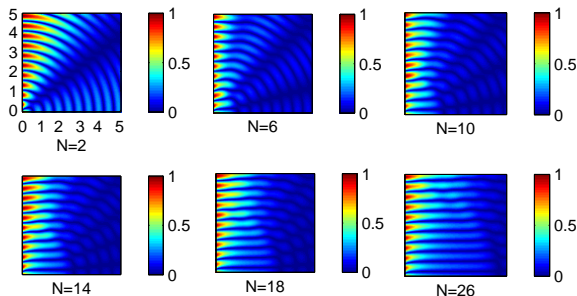
- Trade far-field accuracy for near-field accuracy.
- Simple extension to complex sources.

$|FS|/|PS|$ for $M = 6, 26$:



The velocity error decays $\sim z^{-2}$, so no long range velocity zeros.

FBE from 2D FSE



- No z-axis spike, generally lower error..
- But low orders may still be preferable.

- The driving function is $\frac{\partial S}{\partial z}$ with $S = \text{PS}$
- However, WFS assumes a source free region : FS source generates FS field, but (PS-FS) source will radiate rather than conform to the (PS-FS) field.
- So better to use $S = \text{FS}$.
- And control near-field error with order, same applies to HOA.

Summary

- FS useful for near-field HRTF synthesis.
- Investigated error properties, find truncated expansions beneficial.
- It may be possible to parametrize a range of source expansions for other compromises, but probably none as convenient as the FS.